Core Mathematics C4 For Edexcel Advanced Level

Paper I
Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration.

Full marks may be obtained for answers to ALL questions.

The booklet 'Mathematical Formulae and Statistical Tables', available from Edexcel, may be used.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner.
Answers without working may gain no credit.

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80 Attimore Road
Welwyn Garden City
Herts. AL8 6LP

Tel. 01707 333232

1. A curve is given by the parametric equations

$$x = \sin 2\theta, \quad y = \ln(1 + \cos\theta), \quad 0 \leq \theta < \frac{\pi}{4}.$$

(*a*) Show that the gradient of the curve at the point where $\theta = \frac{\pi}{6}$ is $\sqrt{3} - 2$. **(5)**

(*b*) Find the coordinates of the point where the gradient is zero. **(3)**

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2. Weed is spreading on the surface of a pond so that its area is $A\text{m}^2$ at time t days. It is given that

$$\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{\mathrm{e}^{\frac{1}{10}t}}{A}.$$

Given that $A = 20$ when $t = 0$, solve the differential equation to find the value of A when $t = 20$. Give your answer to 2 significant figures. **(7)**

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3. (*a*) The equation of a curve is

$$\ln y + x^3 - 2x = 0$$

Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = y(2 - 3x^2)$ **(3)**

(*b*) The equation of a curve is

$$\mathrm{e}^x y + y^2 = 9.$$

(i) Find the gradient of the curve at the point (0, 3) **(3)**

(ii) Find the equation of the tangent to the curve at the point (0, 3). **(2)**

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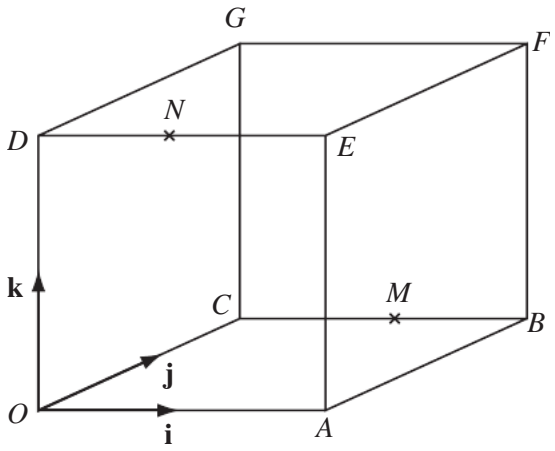
4. (*a*) (i) Express $\cos 2x$ in terms of $\sin x$. **(1)**

(ii) Find $\int \sin^2 x \, dx$ **(2)**

(*b*) Show that $\int_0^{\frac{\pi}{8}} x \sin 2x \, dx = \frac{4-\pi}{16\sqrt{2}}$. **(5)**

5.



The diagram shows a cube *OABCDEFG* with sides of length 2 units. Unit vectors **i, j, k** are directed along *OA, OC, OD* respectively.

The mid-point of *CB* is *M* and the mid-point of *DE* is *N*.

(*a*) Write down the position vectors of the points *M* and *N* **(2)**

(*b*) Write down vector equations for the lines *OM* and *AB* and find the point of intersection of these two lines. **(4)**

(*c*) Calculate the angle between the lines *MN* and *MO*. **(3)**

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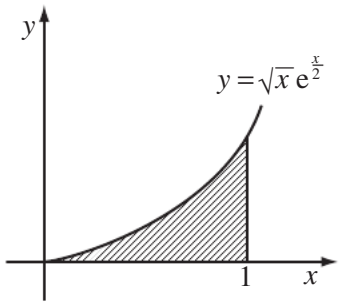
6. (*a*) Find the exact value of $\int_1^2 \frac{2x^2+1}{x}\,\mathrm{d}x$. **(3)**

(*b*) (i) Use integration by parts to show that

$$\int x\mathrm{e}^x\,\mathrm{d}x = \mathrm{e}^x(x-1)+c. \quad \textbf{(3)}$$

(ii) The sketch shows the graph of

$$y = \sqrt{x}\mathrm{e}^{\frac{x}{2}}.$$



The region R, enclosed by the curve and the lines $y = 0$ and $x = 1$, is rotated through four right angles about the x-axis. Find the exact volume of the solid formed. **(4)**

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7. $$\mathrm{f}(x) = \frac{4x+8}{(x+3)(x-1)}, x \neq -3, x \neq 1.$$

(*a*) Express f(x) in partial fractions. **(3)**

(*b*) Obtain the first 3 terms in the expansion of f(x) in ascending powers of x. **(4)**

(*c*) State the range of values of x for which the above expansion is valid. **(1)**

(*d*) Work out f′(x) and prove that f′(x) < 0 for all values of x in the domain. **(3)**

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8. The curve C has equation $y = \dfrac{x}{1+x^2}$.

(a) Find the coordinates of the turning points of C. **(5)**

(b) Determine the nature of each of the turning points. **(3)**

(c) Sketch the curve C. **(3)**

(d) Find the area enclosed by the curve and the lines $y = 0$ and $x = 2$. **(3)**

END **TOTAL 75 MARKS**